

Interpolation Methods for GTD Analysis of Shaped Reflectors

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It is often required to find "smooth" analytic representations for antenna reflector surfaces which are prescribed only by discretized data obtained by various synthesis methods. Frequently the data are distributed in a nonuniform grid and contain noise. The "smoothness" required is to C_1 for physical optics diffraction analysis and to C_2 for Geometrical Theory of Diffraction (GTD) analysis. Furthermore, the GTD analysis approach requires a surface description which returns data very rapidly. Two methods of interpolation, the global and the local methods, are discussed herein. They each have advantages and disadvantages – usually complementary. These characteristics are discussed and examples are presented.

I. Introduction

The diffraction analysis of reflector surfaces which are described only at a *discrete* set of locations usually leads to the requirement of an *interpolation* to determine the surface characteristics over a continuum of locations. Such discretized surface descriptions can come about from a set of point measurements for example. Another common source of such a description is the dual offset shaped reflector synthesis (Refs. 1 and 2), which may involve numerical difference type solutions over a discretized field.

The physical optics analysis of a reflector antenna requires an accurate description of the point characteristics (e.g., $[x, y, z]$ or $[r, \theta, \phi]$) of the surface, and it also requires a reasonably accurate description of the slopes (e.g., $[\partial z/\partial x = z_x, z_y]$ or $[r_\theta, r_\phi]$) at the same points. The GTD analysis requires, further, an accurate knowledge of the second derivatives at the same points. The second derivatives provide the scattered amplitude for both the GO and the diffraction parts

of GTD. Hence the GTD analysis of shaped reflectors with discretized raw data requires an accurate and often time consuming interpolation process.

In dual reflector antennas, it is usually desirable to analyze the subreflector by GTD since more near-zone observation points are required on the main reflector than far field observation points of the main reflector. (We diffract analyze the main reflector by the Jacobi-Bessel method [Refs. 3 and 4].) The interpolation techniques to be described are applicable to both reflectors, but we will describe results found for a shaped subreflector synthesized for high gain.

High gain shaped subreflectors represent a more than average difficult surface to describe because the surface curves more rapidly and often possesses inflection points (Refs. 1 and 5). A profile description of such a dual reflector is shown in Fig. 1. Also illustrated are the projected (on the $[\theta, \phi]$ plane) discretized raw data locations at equal $(\Delta\theta, \Delta\phi)$ increments. In some synthesis methods (Ref. 1), the raw data

consists of r , r_θ , and r_ϕ at each (θ, ϕ) discretized location. However, the derivative data of at least one derivative (in Ref. 1 the r_ϕ derivative) are *unstable* since they are computed by difference techniques which do not permit very small increments.

A method for evaluating the accuracy and stability of a surface description is to compute the “distance” function derivatives D_ϕ and $D_{\phi\phi}$. The distance D is the the distance from the source to a point on the reflector and then to the observation point. When D is a minimum (Fermat) we have a GO or an edge diffraction spectral point. In our evaluation method, we allow the point on the subreflector to vary in position with ϕ (θ fixed). Usually we take $\theta = \theta_{\text{MAX}}$ along the edge of the reflector, although all θ values should be evaluated.

The results for a particular set of raw data (Ref. 5) are shown in Fig. 2. Note the erratic second derivative behavior ($D_\phi = 0$ implies a diffraction spectral point — there are two).

II. Global Interpolation

A global interpolation representation is a closed form or series expression valid over the entire surface. The coefficients of a series expression are found by an integration of the raw data. Since far fewer coefficients are used to describe the surface than raw data points, the integration effectively provides a smoothing of the raw data.

For example, the Jacobi polynomial-sinusoidal (Ref. 3) expansion was found (Ref. 6) to provide a fast converging representation of the offset shaped subreflector discussed. The representation is

$$r(\theta, \phi) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} a_{mn} F_m^n(\theta) \begin{Bmatrix} \cos n\phi \\ \sin n\phi \end{Bmatrix}$$

where the Jacobi polynomial

$$F_m^n(\theta) = \sqrt{2(n+2m+1)} P_m^{(n,0)}(1-2\theta^2) \cdot \theta^n$$

$$P_m^n(s) = \frac{(-1)^m (1-s)^{-n}}{2^m M!} \frac{d^m}{ds^m} \left[\frac{(1-2s)^{m+n}}{(1+s)^{-m}} \right]$$

and the a_{mn} are found by using the orthogonality properties of the expansion functions.

In Fig. 3, the distance function $D_{\phi\phi}$ is found to be perfectly smoothed in the global description of the reflector. In Fig. 4 we observe the convergence of the D_ϕ and the $D_{\phi\phi}$ functions

with $N \times M$ terms. Although $N \times M = 4 \times 4 = 16$ terms are totally adequate for D_ϕ (at the edge), several more terms are required for $D_{\phi\phi}$. Actually the derivatives with respect to θ demand some extra terms. The GTD diffraction pattern for the same shaped subreflector is shown in Figs. 5 and 6. The depicted patterns are taken in the plane of offset of the subreflector which possesses left-right symmetry. A feed with -16 dB taper at the subreflector edge was used. The pattern increases with θ because it is shaped to compensate for the space loss that results when feeding a main offset shaped reflector for high gain. The amplitude is dependent directly upon the second derivatives of the reflector surface.

Figure 5 illustrates the convergence of the pattern with increasing number of global coefficients to represent the surface. We find $4 \times 4 = 16$ terms are adequate except for some cross polarization introduced near $\theta \simeq -3^\circ$. There is no cross polarization with $5 \times 5 = 25$ terms. In Fig. 6, we observe the same GTD pattern obtained for 5×5 global terms and the results obtained directly from the raw data.

The *advantages* of the global representation can be briefly summarized:

- (1) It converges rapidly and uses a small computer core to describe the entire reflector (16-50 real numbers).
- (2) It is analytically smooth through the second derivatives.
- (3) It can be readily used as a synthesis tool with optimization techniques since few terms are involved.

The principle *disadvantage* of the global representation is that it is computationally much slower than closed form expressions such as the hyperboloid formula. In GTD analysis, the search for the GO and edge spectral points (for many observation points) may require many thousands of surface evaluations. Hence a very fast local interpolation (which may use a large set of data and core) is desirable.

III. Local Interpolation

A local interpolation provides a closed form expression for only a small area of the reflector surface. In Fig. 7, we divide the subreflector into three (or more) sectors where each have constant $(\Delta\theta, \Delta\phi)$ discretized data. Each area *segment* is then described by a two-dimensional quadratic surface — locally:

$$z = a_1 + a_2x + a_3y + a_4x^2 + a_5xy + a_6y^2$$

The six coefficients are found locally from six data points and stored for that particular (θ, ϕ) location. Although the second derivatives z_{xx} , z_{xy} , and z_{yy} can be found from the quadratic expression above, we choose to find an *average* $z_{xx\text{AVG}}$,

$z_{xy\text{AVG}}, z_{yy\text{AVG}}$ over the area segment. We thus store nine instead of six real numbers for each segment. We found this averaging necessary because our computer word size (36 bits) would not allow a sufficiently small $(\Delta\theta, \Delta\phi)$ to be used so as to calculate the second derivatives accurately from the quadratic expression above. The averaging method worked successfully.

A quadratic expression was used instead of a higher order local expression because of our required "maximum speed possible" computation. Furthermore, we required our surface to return $(X, Y, Z_x, Z_y; Z_{xx}, Z_{xy}, Z_{yy})$ data from a (θ, ϕ) input. The required computation is fast and simple for a quadratic expression but rapidly becomes more complex as the order of the local surface is increased.

The disadvantages of the local interpolation are essentially the same as the advantages of the global interpolation. In particular, a large core is required for the local interpolation. The advantage is the higher speed and "turn-around" time for computations. The speed is

- (1) Independent of the surface complexity (or the number of terms required by the global method)
- (2) $>20 \times$ global for 50 global terms
 $>10 \times$ global for 25 global terms

The local interpolation does require "smooth" raw data. We found it useful to first obtain a global expression and then do our diffraction analysis rapidly with the local interpolation.

References

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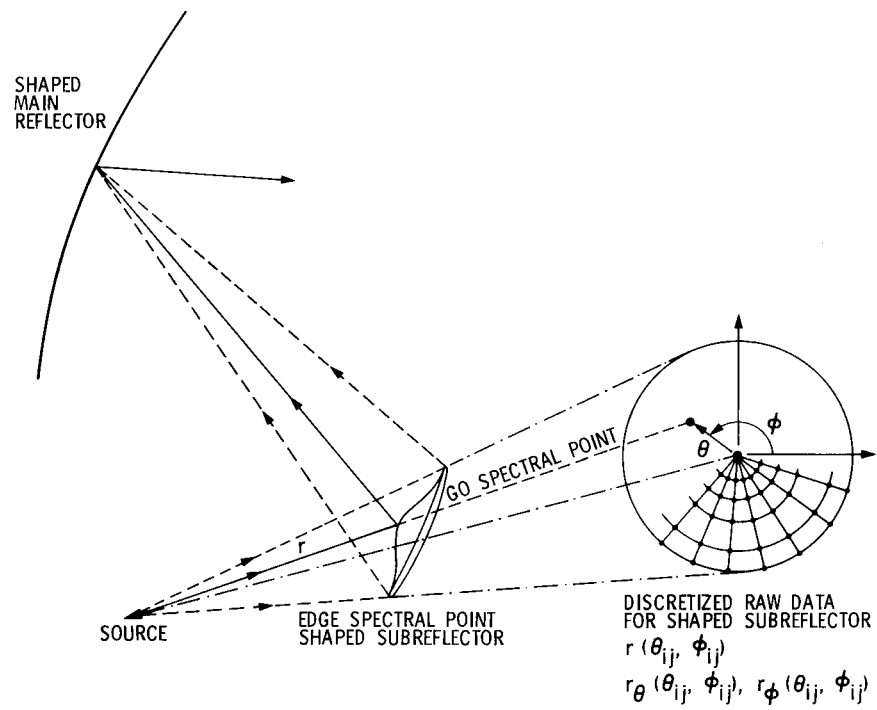


Fig. 1. Dual offset shaped (high gain) reflectors

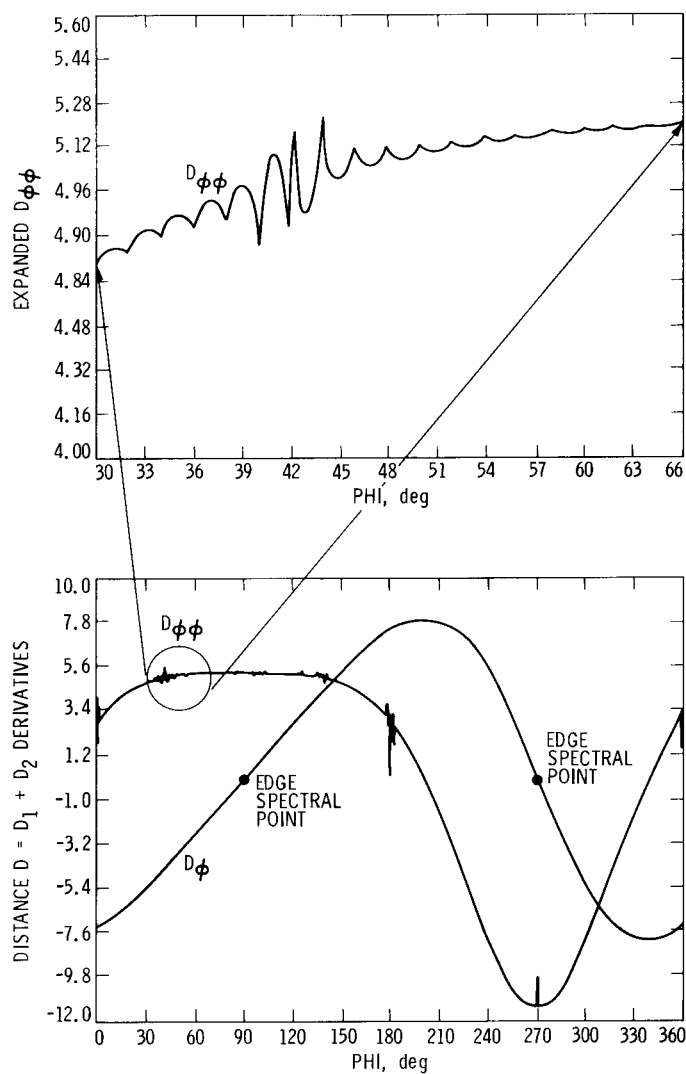


Fig. 2. Distance function derivatives for "raw data"

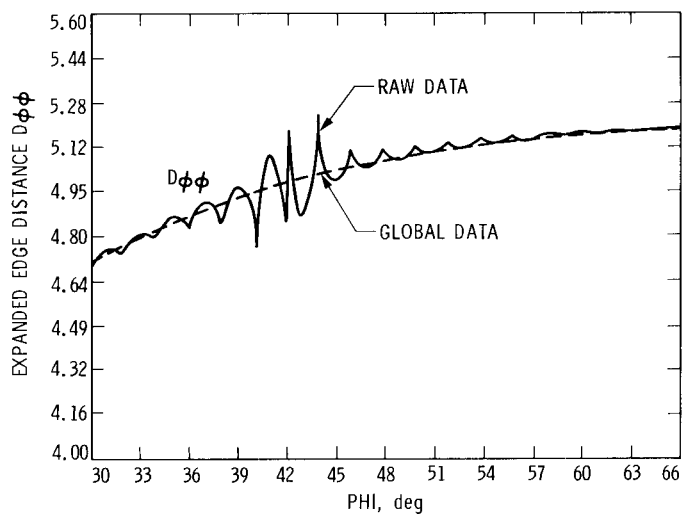


Fig. 3. Distance function $D_{\phi\phi}$ for raw data and global data shaped subreflector (along edge)

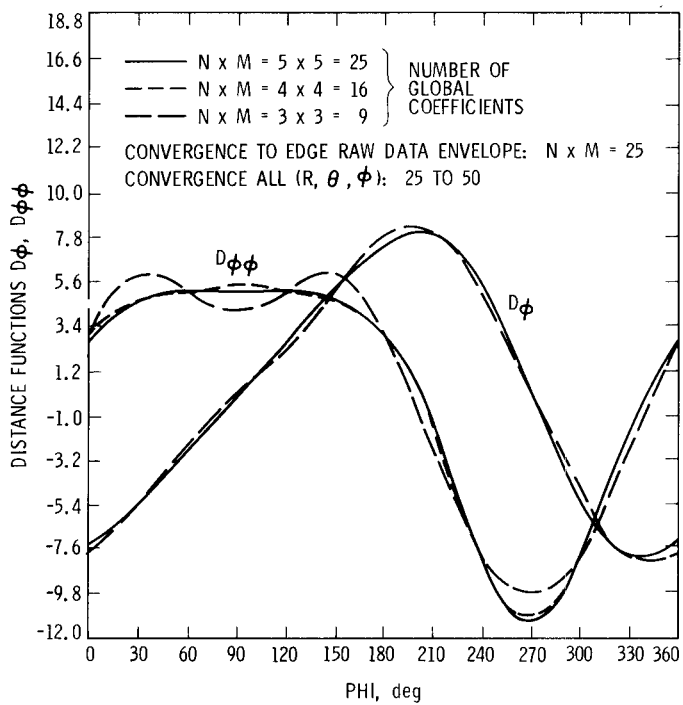


Fig. 4. Distance functions D_{ϕ} , $D_{\phi\phi}$

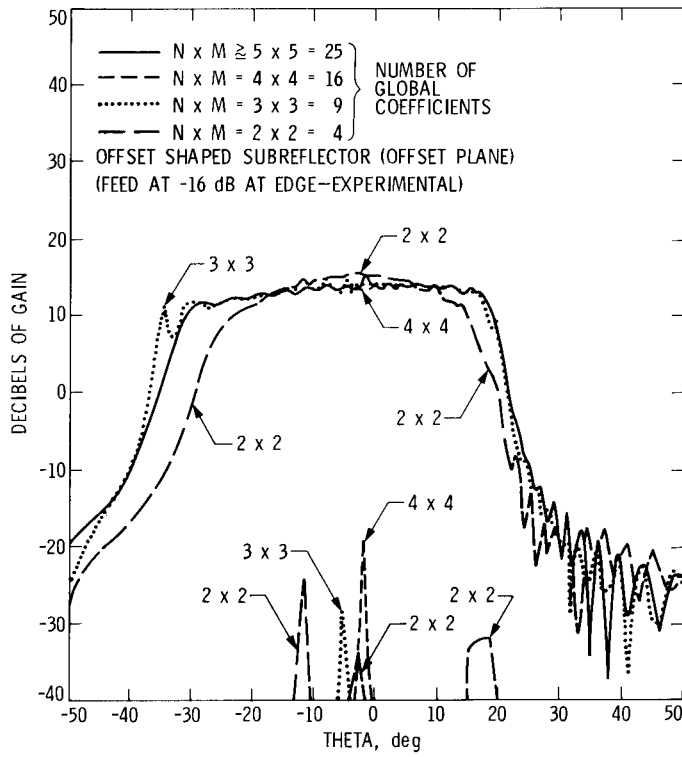


Fig. 5. Scattered field (GO + diffraction) vs global representation

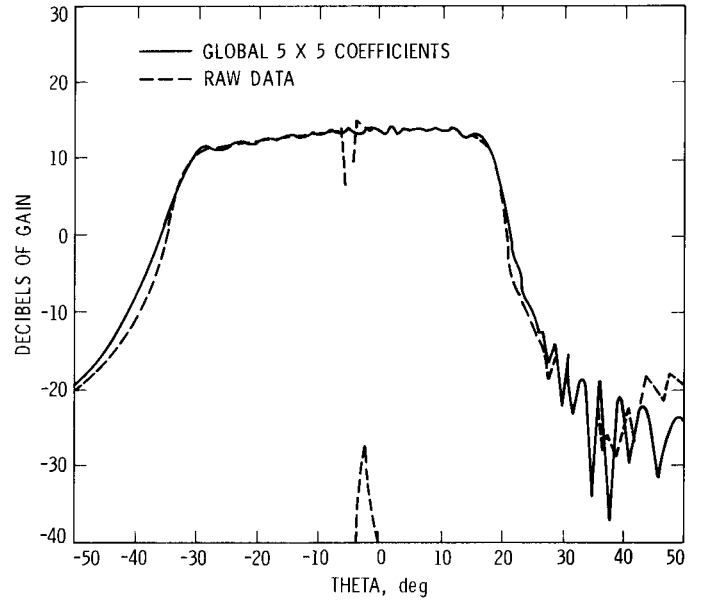


Fig. 6. Scattered field (GO + diffraction) raw data vs global surfaces

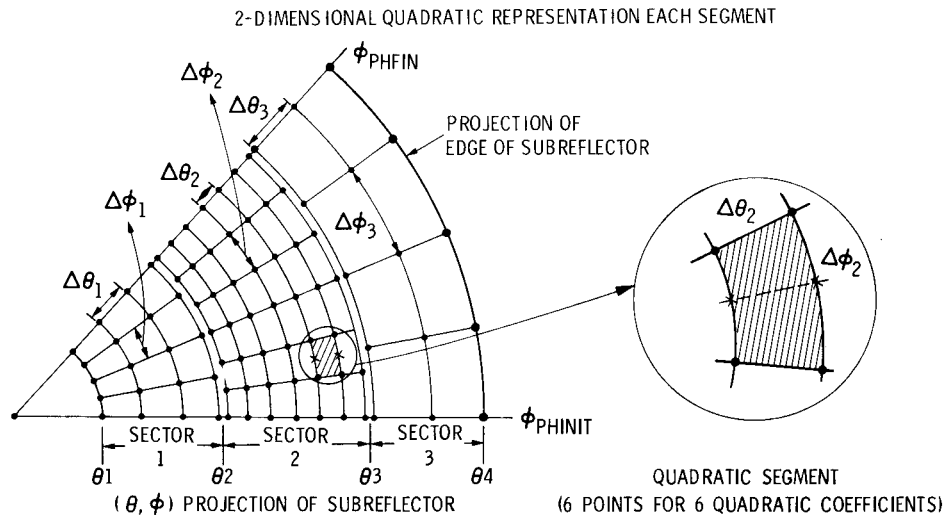


Fig. 7. Local interpolation